# Bose-Einstein Condensate in Artificial Gauge Field

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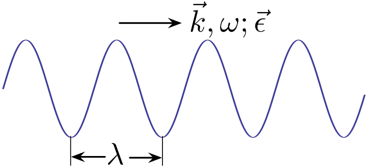
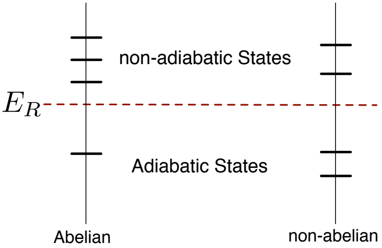
# General Considerations

Goal: Create spatially varying "internal eigenstates" (adiabatic states) of the Hamiltonian. Gauge fields appear in the basis of these adiabatic states.
Method: Using laser (classical field) (+ magnetic field) induced adiabatic states.



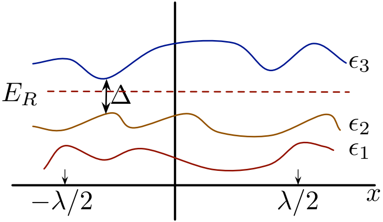

1. Length scales:
laser wave length $\lambda$;
BEC coherence length $\xi$;
inter-particle spacing $d$
2. Typical single particle energy scale: $\frac{\hbar^2 k^2}{2M} \sim E_R$
3. Internal degree of freedom

# Adiabatic Conditions




ADIABATIC CONDITIONS:

1. Single particle energy states $\epsilon_i \ll E_R$;
2. Many-body energy scales $\ll E_R$; (Fermi energy; $\hbar^2/2M\xi^2$,etc.)




Def: $\Delta = \min_x |\epsilon_3 - \epsilon_2|$

3. $\frac{d \ln \epsilon(x)}{dt} \ll \Delta \Rightarrow$

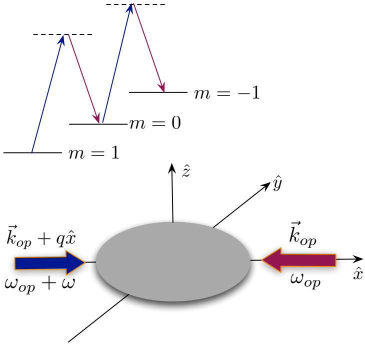
$v \ll \Delta \left(\frac{\partial \ln \epsilon(x)}{\partial x}\right)^{-1} \sim \frac{\Delta}{|\epsilon|}\frac{1}{\lambda}$

Boson: $v \sim \hbar/\xi \Rightarrow \xi \gg \frac{|\epsilon|}{\Delta}\lambda$

Fermion:

$v \sim \hbar/d \Rightarrow d \gg \frac{|\epsilon|}{\Delta}\lambda$

# THE NIST SCHEME 1




## SINGLE PARTICLE HAMILTONIAN:

$$
\begin{aligned}
h(t) &= \frac{\mathbf{p}^2}{2M} + W(t) &(1)\\
W(t) &= -\hbar\Omega_y F_y + \hbar\lambda F_y^2 &(2)\\
&- \frac{\hbar\Omega_R}{2}\left[e^{i(qx-\omega t)}F_+ + h.c.\right]
\end{aligned}
$$

$$
\begin{aligned}
F_+ &\equiv F_z + iF_x &(3)\\
F_- &\equiv F_z - iF_x &(4)
\end{aligned}
$$

$$\hbar\Omega_y = \hbar\Omega_o + Gy \qquad (5)$$

# THE NIST SCHEME 2

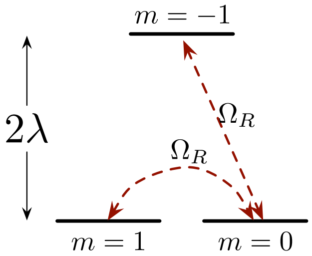
**CONDITION FOR NON-ABELIAN FIELDS:**

STEP 1: Go to rotating frame:

$$W(t) = e^{-iqxF_y}\left(-\overline{\Omega}_y F_y + \lambda F_y^2 - \Omega_R F_z\right)e^{iqxF_y} \tag{6}$$

$$\hbar\overline{\Omega}_y = \hbar\Omega_o - \omega + Gy \tag{7}$$

STEP 2: Setting $G = 0$. Choose $\omega = \lambda + \Omega_o$. Then the states $m = 0$ and $m = -1$ are degenerate and the state $m = -1$ is of energy $2\lambda$ higher.




Provided $\lambda \gg \Omega_R, E_R$, can neglect coupling to $m = -1$ state $\Rightarrow$ effectively two internal states.

# THE NIST SCHEME 3

STEP 3: Transform to spatially dependent basis.

$$\hat{\psi}^\dagger_m \text{ — create state in the lab frame}$$
$$\hat{\phi}^\dagger_m \text{ — create state in the rotating frame}$$

$$\hat{\psi}^\dagger_m = \left(e^{iqxF_y}\right)_{mn} \hat{\phi}^\dagger_n \tag{8}$$

THE SINGLE PARTICLE HAMILTONIAN:

$$H_{mn} = \frac{\hbar^2}{2M}\left[\frac{\nabla}{i} + \hat{\mathbf{x}}q \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\right]^2 + \hbar \begin{pmatrix} 0 & \frac{\Omega_R}{\sqrt{2}} \\ \frac{\Omega_R}{\sqrt{2}} & 0 \end{pmatrix}. \tag{9}$$

The full Hamiltonian (including chemical potential term):

$$\hat{\mathcal{K}} = \int \left[\hat{\phi}^\dagger_m H_{mn} \hat{\phi}_n + \frac{1}{2}\hat{n}_m g_{mn} \hat{n}_n + (V - \mu)\hat{n}\right] \tag{10}$$

$g_{mn}$ — interacting matrix elements in the rotating basis.

## Properties of $H_{mn}$ 1

(1). If $\chi_m$ is the solution of the Hamiltonian $H_{mn}$ with energy $E$, then

$$\chi_n = e^{i\gamma} e^{-iqx} \left(\tau_1\right)_{nm} \chi_m^* \tag{11}$$

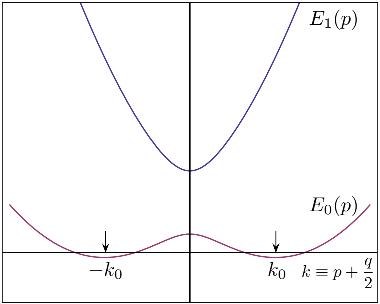
is also an eigenstate, with the same energy $E$. $\gamma$ is an arbitrary phase factor.
(2). $\chi_m^{(p)}(x) = e^{ipx}\widetilde{\chi}_m$, $\widetilde{\chi} \equiv \binom{u}{v}$, $\ell^2 \equiv \frac{M\Omega_R}{\sqrt{2}\hbar}$

$$\frac{\hbar^2}{M}\left(\frac{k^2+Q^2}{2} + kQ\tau_2 + \ell^2\tau_1\right)\begin{pmatrix} u \\ v \end{pmatrix} = E_p \begin{pmatrix} u \\ v \end{pmatrix}, \tag{12}$$

$$E_{1(0)}(p) = \frac{\hbar^2}{M}\left(\frac{k^2+Q^2}{2} + (-)\sqrt{(kQ)^2+\ell^4}\right) \tag{13}$$

$$Q = q/2; k = p + Q \tag{14}$$

# Properties of $H_{mn}$ 2




Def: $\frac{M\Omega_R}{\sqrt{2}\hbar Q^2} \equiv \sin\theta$;
The lowest energy states are degenerate @

$$
\begin{aligned}
k_o &= Q\sqrt{1-\frac{\ell^4}{Q^4}} = \frac{q}{2}\cos\theta; \\
E_0(p_\pm) &= -\frac{m(\hbar\Omega_R)^2}{\hbar^2 q^2}; \\
p_\pm &= \pm k_o - q/2
\end{aligned}
$$

$$
\widetilde{\chi}^{(p_+)} = \begin{pmatrix} i\sin\frac{\theta}{2} \\ \cos\frac{\theta}{2} \end{pmatrix}, \quad \widetilde{\chi}^{(p_-)} = \begin{pmatrix} i\cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} \end{pmatrix}. \tag{15}
$$

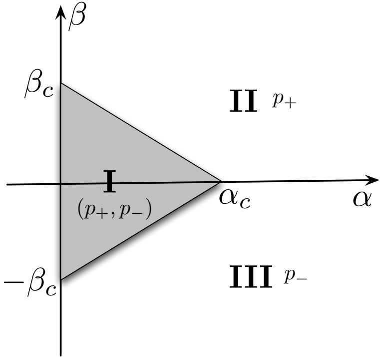
Note: $\widetilde{\chi}^{(p_+)\dagger}\widetilde{\chi}^{(p_+)} = \sin\theta \neq 0.$

# Structure of the Condensate

The Gross-Pitaevskii ansatz can be written as:

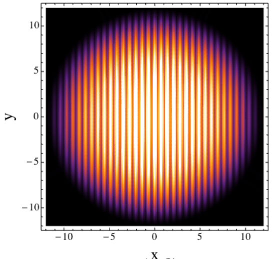
$$\Phi_m(x) = A_+\chi_m^{(p_+)}(x) + A_-\chi_m^{(p_-)}(x). \tag{16}$$

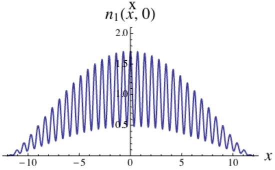
Our task is to fix $A_+$ and $A_-$, which in general are complex amplitudes.
$\Rightarrow$ minimize the GP energy functional $\hat{\mathcal{K}}$.




$$\begin{aligned}
\alpha &\equiv (g_{10})/g; \\
\beta &\equiv (g_{11} - g_{00})/g; \\
g &\equiv (g_{11} + g_{00})/2; \\
\alpha_c &= \frac{2 - \tan^2\theta}{2 + \tan^2\theta} > 0; \\
\beta_c &= \cos\theta(2 - \tan^2\theta)
\end{aligned}$$

# Stripe Structure




In the region **I**, $(p_+, p_-)$:

$$\Phi_m = \sqrt{\frac{\mu(\mathbf{r})-E_o}{G_o}}\left[a_+^o e^{ip_+x}\begin{pmatrix} i\sin\frac{\theta}{2} \\ \cos\frac{\theta}{2}\end{pmatrix} + e^{i\gamma}a_-^o e^{ip_-x}\begin{pmatrix} i\cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}\end{pmatrix}\right] \qquad (17)$$

$$\mu(\mathbf{r}) = \mu - V(\mathbf{r}) \qquad (18)$$

$$\alpha = \frac{1}{4}\alpha_c, \beta = \frac{1}{4}\beta_c$$

# Thank you!