Heat and Spin Transport of Cold Atomic Fermi Gases

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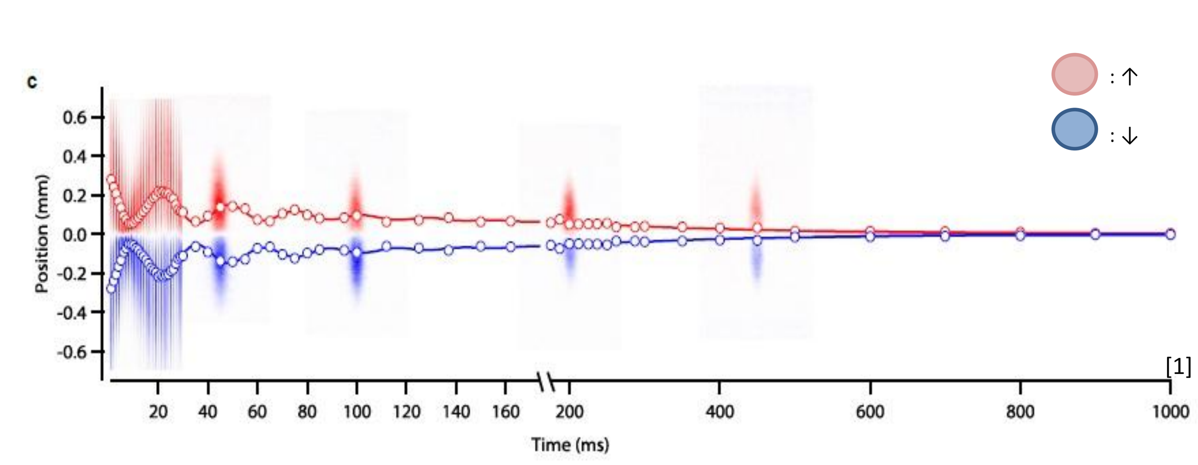
Abstract

Motivated by recent experiments measuring the spin transport in ultracold unitary atomic fermi gases [1,2], we explore the theory of spin and heat transport in a spin-polarized atomic fermi gas in three dimensional space. We develop estimates of spin and thermal diffusivities and discuss magnetocaloric effects, the spin Seebeck and the spin Peltier effects. We quantitatively estimate the spin Seebeck effect using Boltzmann kinetic equation in classical regime. We propose a schematic experimental procedure to detect the spin Seebeck effect which does not require knowledge of exact equations of state.

Transport Measurement[1,2]



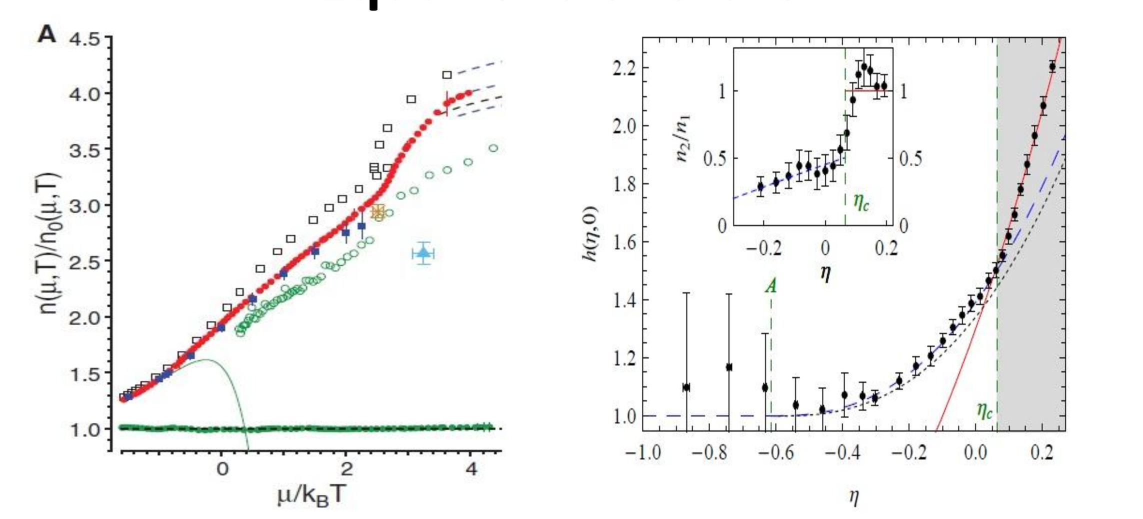
Early -- Collective motions:
- Nonuniform pressure (out of mechanical equilibrium)
- Excited collective modes
- Damping fast

(Ref: Taylor et al, Phys. Rev. A 84, 063622 (2011) & more)

Later -- Diffusive relaxation:
- **Uniform pressure (mechanical equilibrium)**
- Damping slow
- Observe space-time dependence of densities
- Measure spin diffusivity

[1] Sommer et al, *Nature* **472**, 201 (2011) [2] Sommer et al, NJP **13**, 055009 (2011)

Equations of State



Unpolarized Gas [3,4] Polarized Gas [5,6]

Once we know the Equations of State
$\Rightarrow \nabla T$, $\nabla\mu_\uparrow, \nabla\mu_\downarrow$ from $\nabla n_\uparrow, \nabla n_\downarrow$ **at mechanical equilibrium (without optical lattice – Galilean invariance)**
$\Rightarrow$ Able to observe **Thermal diffusion from ∇T**

[3] Ku, et al, *Science* **335**, 563 (2012)
[4] van Houcke, et al *Nature Phys.* **8**, 366 (2012)
[5] Nascimbene, et al, *Nature* **463**, 1057 (2010)
[6] Lobo et al, PRL **97**, 200403 (2006)

Transport Coefficients

$$\begin{pmatrix} \mathbf{J}_h \\ \mathbf{J}_s \end{pmatrix} = -\begin{pmatrix} D_T C_p & Pe \\ Se & D_s\chi_s \end{pmatrix}\begin{pmatrix} \nabla T \\ \nabla(\mu_\uparrow-\mu_\downarrow) \end{pmatrix}$$

C_p : Heat capacity
χ_s : Spin susceptibility

Spin diffusivity : D_s

Thermal diffusivity : D_T

Spin Peltier effect (Pe) : $\nabla(\mu_\uparrow-\mu_\downarrow) \Rightarrow \mathbf{J}_h$

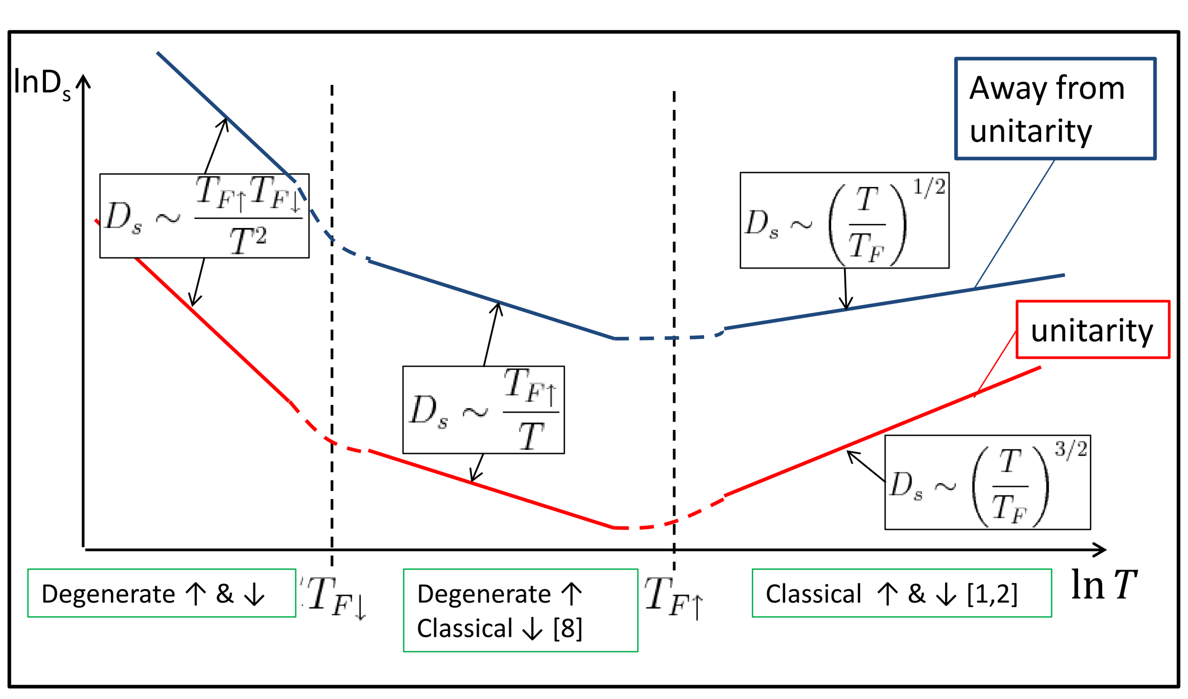
Spin Seebeck effect (Se) $\nabla T \Rightarrow \mathbf{J}_s$

MagnetoCaloric Effects [7]

Onsager Relation : $Pe = T\,Se$

Estimates of Diffusivities

At high polarization : $D_s \sim v_\downarrow^2 \tau_\downarrow$



$$D_T \sim v_\uparrow^2\tau_\uparrow\ ,\ \tau_\uparrow \sim \left(\frac{n_\uparrow}{n_\downarrow}\right)\left(\frac{v_\downarrow^2}{v_\uparrow^2}\right)\tau_\downarrow \Rightarrow D_T \sim \frac{n_\uparrow}{n_\downarrow}D_s > D_s$$

[7] Uchida et al, *Nature* **455** 07321 (2008) [8] Bruun et al, *PRL.* **100** 240406 (2008)

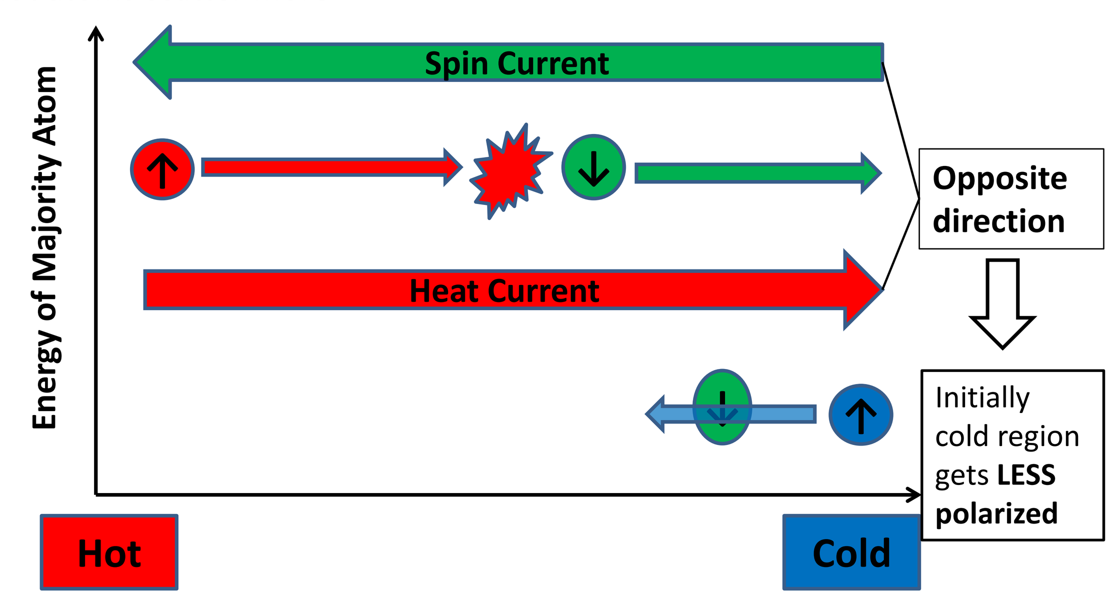
Qualitative Magnetocaloric Effects

- If $n_\uparrow = n_\downarrow$, symmetry dictates: $Pe = T\,Se = 0$ [9]
- Spin Seebeck effects at $n_\uparrow > n_\downarrow$:

(1) Away from Unitarity

Scattering rate : $\frac{1}{\tau_\downarrow} \sim n_\uparrow\sigma|\mathbf{v}_\uparrow - \mathbf{v}_\downarrow| \sim n_\uparrow a^2|\mathbf{v}_\uparrow - \mathbf{v}_\downarrow|$

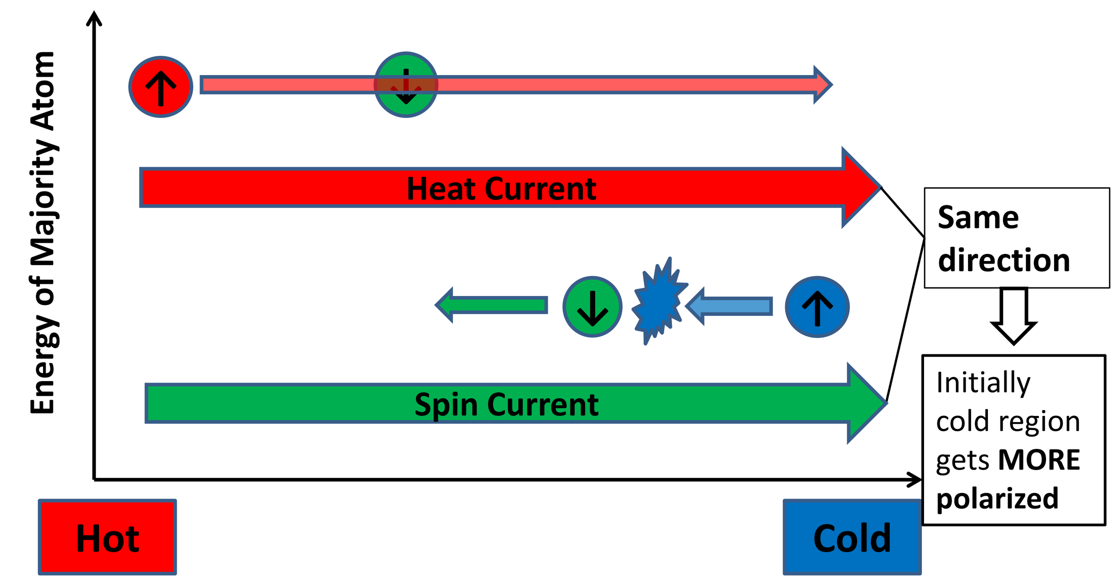
Hot atoms scatter more



(2) Near Unitarity

Scattering rate : $\frac{1}{\tau_\downarrow} \sim n_\uparrow\sigma|\mathbf{v}_\uparrow - \mathbf{v}_\downarrow| \sim \frac{n_\uparrow}{|\mathbf{v}_\uparrow - \mathbf{v}_\downarrow|}$

Cold atoms scatter more.



[9] Exception : See Wong et al, arXiv: 1203.3745v1

Quantitative Approaches

- Boltzmann equation with polynomial approximation :

$$\frac{\partial f_\sigma}{\partial t} + \mathbf{v}_\sigma\cdot\nabla_\mathbf{r} f_\sigma + \mathbf{F}_\sigma^{ext}\cdot\nabla_\mathbf{k} f_\sigma = \left(\frac{\partial f_\sigma}{\partial t}\right)_{coll}$$

- Kubo formula :

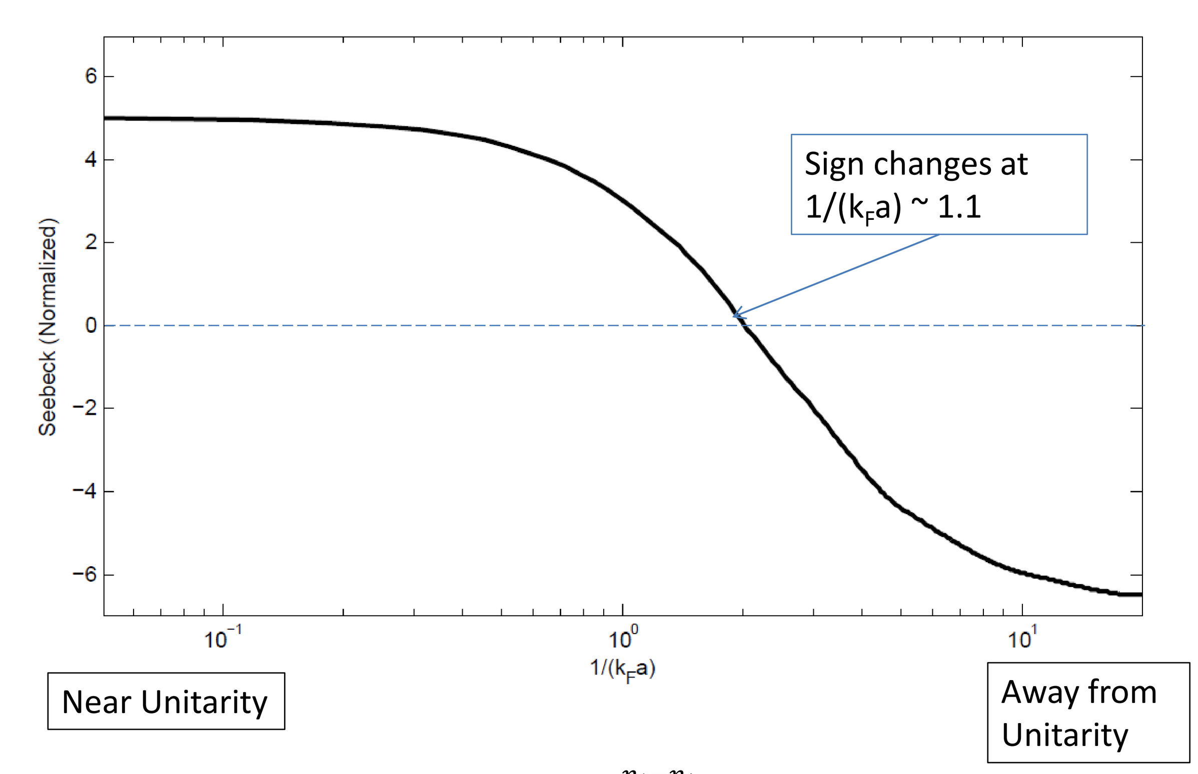
$$\Gamma = \frac{1}{VT}\int_0^\infty dt < \mathbf{J}(t)\cdot\mathbf{J}(0) >$$

- Perturb exactly solvable model, Maxwellian atoms :

$$\sigma \sim \frac{1}{|\mathbf{v}_\uparrow - \mathbf{v}_\downarrow|}$$

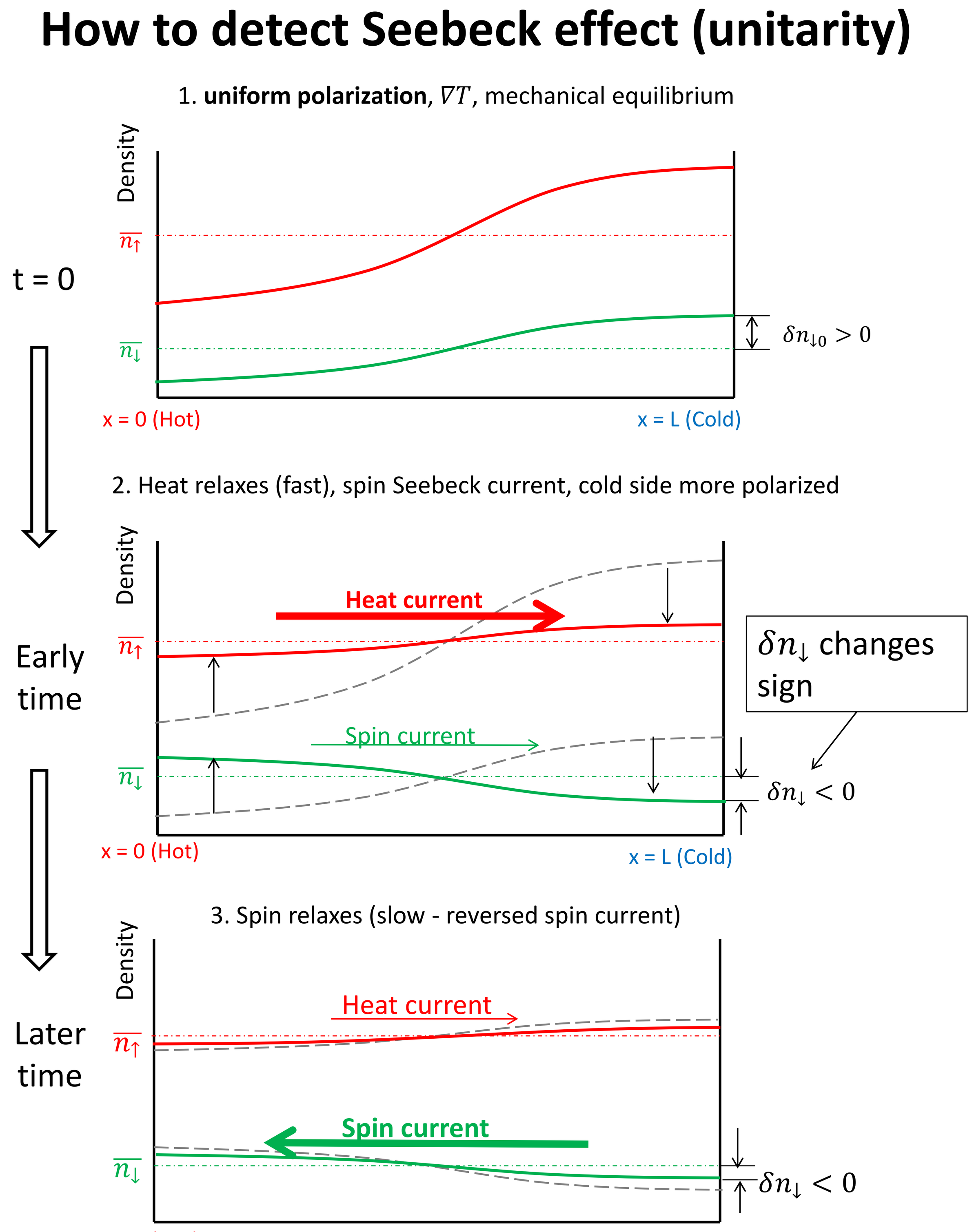
$\Rightarrow$ **They ALL give the same qualitative physics**

Spin Seebeck effect from Boltzmann Eqs.

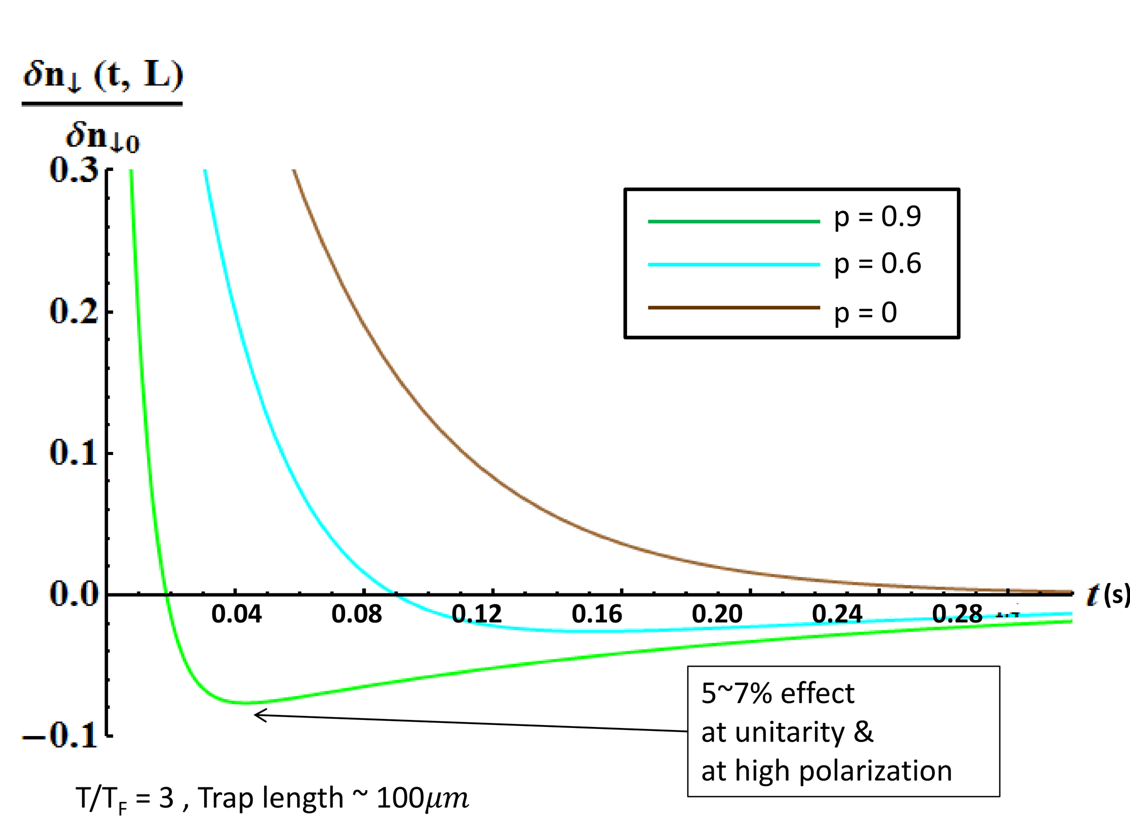


Polarization = $\frac{n_\uparrow - n_\downarrow}{n_\uparrow + n_\downarrow} = 0.8$

How to detect Seebeck effect (unitarity)



Signatures of spin Seebeck effect



- **Nonmonotonic** relaxation of density deviation
  - Unitarity : $\delta n_\downarrow$
  - Away from unitarity : $\delta n_\uparrow$
- Small effect : 5~7 %
- **Apparent even at high T**
- **Qualitative effect : Sign change**

Quantum regime – polarized Fermi liquid

- T << $T_{F\uparrow}$ & $T_{F\downarrow}$
- High enough polarization not to form superfluid
- Transport of **quasi particles**
- **In medium scattering**
  - Scattering depends on **center of mass momentum**

=> Stay Tuned!

Summary & Outlook

- Description of transport coefficients of diffusive currents
- Qualitative & quantitative estimates of magnetocaloric effects at high temperature
- Quantitative estimates in a harmonic trap?
- Transport with optical Lattice?
- Transport at superfluid / normal fluid interface?