Levitation in the weak-field Quantum Hall Effect

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The Quantum Hall Effect

When a strong perpendicular magnetic field is applied to a 2D electron gas the transverse resistivity, ρ_{xy}, takes on quantized values h/ne^2, where $n \in \mathbb{Z}$ as shown below.

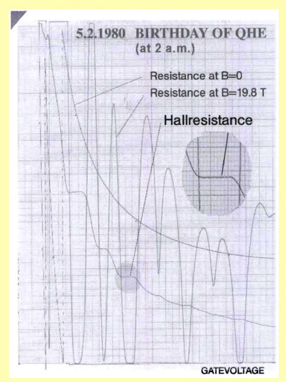


Figure 2: Hall resistance and longitudinal resistance (at zero magnetic field and at $B = 19.8$ Tesla) of a silicon MOSFET at liquid helium temperature as a function of the gate voltage. The quantized Hall plateau for filling factor 4 is enlarged.

A classical calculation only predicts an inverse relationship with the charge carrier density, but the quantization occurs because when the Fermi energy is between two Landau leves ($E_n = \hbar\omega_c(n + 1/2)$) the states are localised and hence do not contribute to transport phenomena.

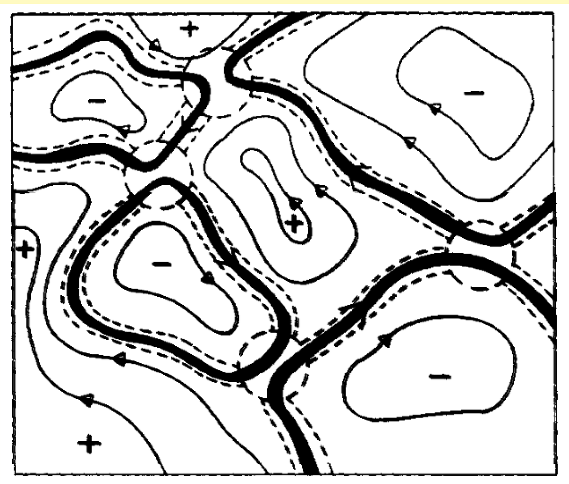
Role of disorder

The presence of disorder in the system is what causes the localization of some states. We know that in the absence of a magnetic field all states would be localised, so what creates these extended states at the centre of each Landau level?

The Microscopic Mechanism

In a strong magnetic field when the cyclotron radius is much smaller than the length scales associated with the disorder potential, classically electrons will perform a 'guiding centre' motion along the equipotential lines of the potential.

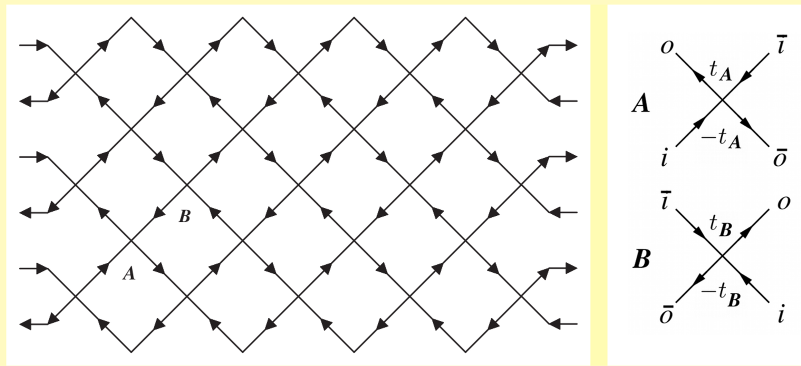
At energies above and below a critical value (energy of the underlying Landau level) we have several localised orbits. However, close to the critical energy these semiclassical orbits come close enough at points that allow tunneling from one orbit to another. This is what leads to the extended state we observe at the Integer Quantrum Hall transition.



Sketch of a typical potential, V(x,y). Full curves represent equipotentials and arrows give direction of guiding centre motion; + and - denote maxima and minima. Heavy curves indicate contours at a particular potential V_0. Portions of these contours are enclosed in strips and circles (dashed lines) which will correspond to links and nodes of the network model.

The CC Network Model

This physical picture above can be approximated quite well by the simple Chalker-Coddington (CC) network model.



In this model the electrons are restricted on a regular lattice with a corresponding discrete evolution operator given as follows:

- There is a $SU(2)$ scattering matrix at the nodes given by: $\begin{pmatrix} \sqrt{1-t^2} & t \\ -t & \sqrt{1-t^2} \end{pmatrix}$
- There is a random phase, $e^{i\phi}$, associated with traversing a link

The scattering matrix simulates the tunneling between plaquettes, and the random phase is the accumulated Aharonov Bohm phase of the guiding centre motion.

The Spin Chain & Sigma Model

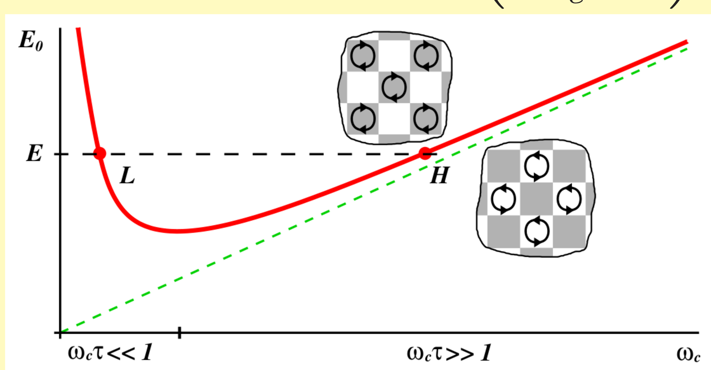
In the limit $t \to 0$ it is possible to take a continuum limit and obtain a 1D Hamiltonian for a staggered superspin chain, with alternating couplings proportional to t_A^2 and t_B^2.

We can take a continuum limit in the other direction, which gives us a Nonlinear-Sigma model (NLSM) with a θ-term. This NLSM is known to have a critical point when the θ-term is an odd multiple of π, which happens at $t_A = t_B$.

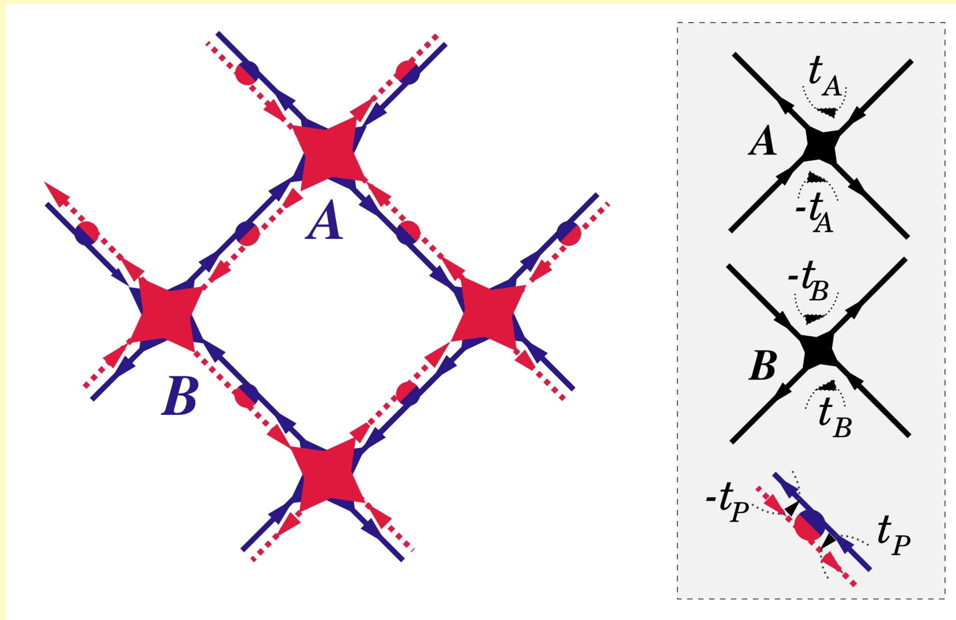
What about low magnetic field?

RG arguments suggest that the energy of the n^{th} extended state goes as:

$$E_n = \hbar\omega_c(n+1/2)\left(\frac{1+\omega_c^2\tau^2}{\omega_c^2\tau^2}\right)$$



Microscopic Model for the Low Magnetic Field Critical states



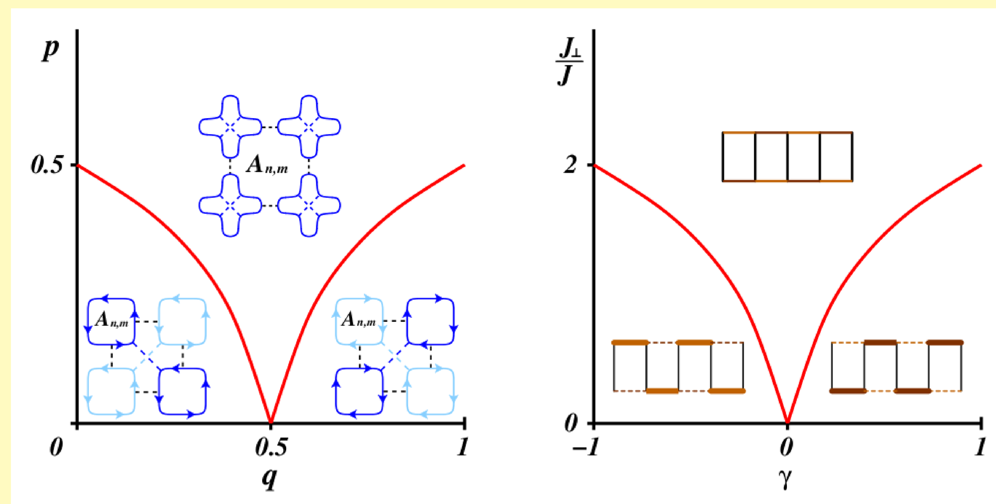
Consider this network model to explain the low field critical Quantum Hall state. Since we are low field the motion is no longer unidirectional. In this network we have:

- Turning probabilities, $t_A = \lambda q, t_B = \lambda(1-q)$, describe the effect of the weak magnetic field in slightly biasing the left/right turning probabilities, with $q = 1/2$ denoting zero field.
- The back scattering amplitude p decreases monotonically with increasing energy
- We also associate random Aharonov Bohm phases to the links.

Superspin Ladder

We introduce bosons on the links in addition to the fermions to enable us to average over the disordered $U(1)$ field. Once this disorder averaging is carried out it imposes a condition on the allowed numbers of advanced/retarded fermions/bosons on each link. This allows us to recast the creation/annihilation operators for the particles on the links in terms of $U(1,1|2)$ superspin. In the limit $t_x, \lambda \to 0$ we can take a continuum limit in the vertical direction and write down a 1D hamiltonian for a spin ladder:

$$\begin{aligned} H_{1D} \;=\; & -\mathrm{Str}\sum_k J_\perp\left(\mathcal{S}_{-,2k}\bar{\mathcal{S}}_{+,2k} + \bar{\mathcal{S}}_{-,2k-1}\mathcal{S}_{+,2k-1}\right) + J(1+\gamma)\left(\mathcal{S}_{-,2k}\bar{\mathcal{S}}_{-,2k+1} + \mathcal{S}_{+,2k-1}\bar{\mathcal{S}}_{+,2k}\right) \\ & + J(1-\gamma)\left(\mathcal{S}_{-,2k}\bar{\mathcal{S}}_{-,2k-1} + \mathcal{S}_{+,2k+1}\bar{\mathcal{S}}_{+,2k}\right) \end{aligned} \tag{1}$$



- The above phase diagram was obtained by taking the continuum limit of the spin ladder to a sigma model.
- The sigma model also confirms that the critical state obtained here is in the same universality class as the CC network model, suggesting that this model might indeed be a correct description of the levitating delocalised states in a low magnetic field.

References

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